PRE-CALCULUS & TRIGONOMETRY SUMMER ASSIGNMENT

Part I: Laws of Exponents

Laws of Exponents		
product	$a^m \cdot a^n = a^{m+n}$	$2^2 \cdot 2^3 = (2\cdot2)(2\cdot2\cdot2) = 2^5$
quotient	$\frac{a^m}{a^n} = a^{m-n}$	$\frac{2^3}{2^2} = \frac{2\cdot2\cdot2}{2\cdot2} = 2^{3-1} = 2$
power	$(a^m)^n = a^{m\cdot n}$	$(2^2)^3 = (2\cdot2\cdot2)(2\cdot2\cdot2)(2\cdot2\cdot2) = 2^6$
inverse	$a^{-1} = \frac{1}{a}$	$2^{-1} = \frac{1}{2}$ (this is a definition)

Error		Correct
$x^2 \cdot y^3 \neq (xy)^5$	the two numbers multiplied do not have the same base.	no way to simplify
$2x^{-1} \neq \frac{1}{2x}$	Unless there are ( ), around 2x, the exponent applies only to the x	$2x^{-1} = \frac{2}{x}$
$x^3 \cdot x^5 \neq x^{15}$	See the product law above. Multiplication → add exponents	$x^3 \cdot x^5 = x^8$
$(x^3)^5 \neq x^8$	See the power law above. When raising a power to a power, multiply exponents.	$(x^3)^5 = x^{15}$

The zero exponent rule: $a^0 = 1,\ a \neq 0$

Simplify the expression using the laws of exponents. There should be no negative exponents when completely simplified.

1. $x^5 \cdot x^2$

2. $y^3 \cdot y \cdot y^4$

3. $2b^4 \cdot 3b^{-4}$

4. $\frac{5x^4}{x^9}$

5. $\frac{2c^5}{4c^3}$

6. $\frac{b^3 \cdot b^4}{b^2}$

Distributive Property of Exponents:

Property	Example
$(xy)^a = x^a y^a$	$(x^2y^3)^2 = (x^2)^2(y^3)^2 = x^{2\cdot2}y^{3\cdot2} = x^4y^6$
$\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$	$\left(\frac{x^{2a}}{y^{3b}}\right)^3 = \frac{(x^{2a})^3}{(y^{3b})^3} = \frac{x^{2a\cdot3}}{y^{3b\cdot3}} = \frac{x^{6a}}{y^{9b}}$

7. $(3x^2)^4$

8. $(2ab)^5$

9. $(x^2y^4m^3)^8$

10. $(2x^3y^6)^6$

11. $p^2 \cdot (p^5)^2$

12. $\left(\frac{3x^3}{4y^4}\right)^2$

Putting it all together:

25) $\frac{3m^{-4}}{m^{3}}$

26) $\frac{2x^{4}y^{-4}z^{-3}}{3x^{2}y^{-3}z^{4}}$

29) $\frac{4m^{4}n^{3}p^{3}}{3m^{2}n^{2}p^{4}}$

30) $\frac{3x^{3}y^{-1}z^{-1}}{x^{-4}y^{0}z^{0}}$

Part II: Factoring

Find the GCF for: $30x^3 + 5x^2 - 25x$

Step 1: Look at the coefficients

Ask yourself: Is there a number that I can divide 30, 5, and 25 by evenly? Yes, 5!
(30/5 = 6) (5/5=1) (-25/5 = -5)

Step 2: Look at the variable(s).

Ask yourself: Can I factor out a variable from EVERY term? Yes! Each term has at least one x, therefore, I can factor out x.

Step 3: Identify the GCF:
The GCF for this polynomial is: 5x. (You can divide every term by 5x evenly (without creating a fraction).

Example of factoring by GCF

Take out the GCF	EX: $15xy^2 - 10x^3y + 25xy^3$
How: Find what is in common in each term and put in front. See what is left over. Check answer by distributing out.	Solution: $5xy(\ 3y - 2x^2 + 5y^2\ )$

Factor by GCF:

1. $7ab - 35a^2b$
2. $-3a^2b + 6a^3b^2$
3. $-5x^2 - 5x^3 - 15x^4$
4. $30b^9 + 5ab - 15a^2$

FACTOR BY GROUPING

1. Group the first two terms, and group the second two terms.
2. Factor out the GCF of both pairs.
3. Now the GCF is (x+5)
4. Bring down what is left.
5. Check by multiplying the binomials.

$$\underbrace{2wx+10w}_{\text{factor}}+\underbrace{7x+35}_{\text{factor}}$$

$$= 2w(x+5)+7(x+5)$$

$$= (x+5)(2w+7) \text{ done!}$$

Factor by grouping:

1. $8r^3 - 64r^2 + r - 8$

2. $12p^3 - 21p^2 + 28p - 49$

3. $12x^3 + 2x^2 - 30x - 5$

4. $6v^3 - 16v^2 + 21v - 56$

5. $63n^3 + 54n^2 - 105n - 90$

Factor the trinomial (using any method)

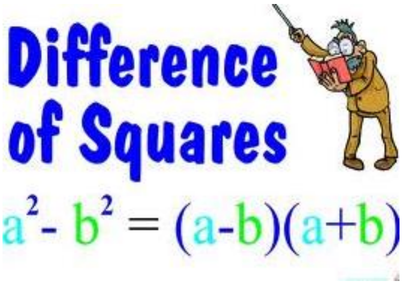
1. $b^2 + 8b + 7$

2. $n^2 - 11n + 10$

3. $m^2 + m - 90$

4. $7x^2 - 45x - 28$

5. $2b^2 + 17b + 21$



Example:

$9x^2 - 64$

$a = 3x \text{ and } b = 8$

$(3x)^2 = 9x^2 \text{ and } 8^2 = 64$

$(3x + 8)(3x - 8)$

1. $r^2 - 4$

2. $9x^2 - 1$

3. $9v^2 - 121$

4. $1 - 36x^2$

5. $144 - x^2$

6. $m^2 + 100$

Solving by Quadratic Formula

Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The solutions of some quadratic equations are not rational, and cannot be factored. For such equations, the most common method of solution is the quadratic formula. The quadratic formula can be used to solve ANY quadratic equation, even those that can be factored.
Be sure you know this formula!!!

Note: The equation must be set equal to zero before using the formula.

Example:

$$x^2 - 5x - 3 = 2$$

$$x^2 - 5x - 5 = 0$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-5)}}{2(1)}$$

$$x = \frac{5 \pm \sqrt{25+20}}{2} = \frac{5 \pm \sqrt{45}}{2}$$

$$= \frac{5 \pm 3\sqrt{5}}{2}$$

Solutions:

$$x = \frac{5+3\sqrt{5}}{2}; \quad x = \frac{5-3\sqrt{5}}{2}$$

As decimal values:

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(5+3√(5))/2
        5.854101966
(5-3√(5))/2
       -.8541019662
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Solve using the Quadratic Formula:

1. $n^2 + 9n + 11 = 0$

2. $5p^2 - 125 = 0$

3. $2x^2 - 5x - 1 = 0$

Solving by Factoring

Some quadratic equations can be solved by factoring. Set the equation equal to zero and factor.

Example 1:
Solve by factoring:

$x^2 - 12x + 27 = 0$

$(x-9)(x-3) = 0$

$x - 9 = 0 \quad x - 3 = 0$

$x = 9 \quad x = 3$

Example 2:
Solve by factoring:

$x^2 - 64 = 0$

$(x+8)(x-8) = 0$

$x + 8 = 0 \quad x - 8 = 0$

$x = -8 \quad x = 8$

Example 2:
Solve by factoring:

$2x^2 + 7x + 3 = 0$

$(2x+1)(x+3) = 0$

$2x + 1 = 0; \quad x + 3 = 0$

$x = -\frac{1}{2}; \quad x = -3$

Solve by Factoring:

1. $6n^2 + 5n - 25 = 0$

2. $2x^2 - 11x - 21 = 0$

3. $5v^2 + 3 = -16v$

4. $2n^2 + 13n + 19 = 4$

Solving by Competing the Square

Quadratic equations can be solved by completing the square.

Example:

$x^2 + 4x - 4 = 0$ $x^2 + 4x = 4$	Keep all terms containing x on one side. Move the constant to the right.
$x^2 + 4x + \square = 4 + \square$	Get ready to create a perfect square on the left. Balance the equation.
$x^2 + 4x + \boxed{4} = 4 + \boxed{4}$ $x^2 + 4x + 4 = 8$	Take half of the x-term coefficient and square it. Add this value to both sides.
$(x+2)^2 = 8$	Simplify and write the perfect square on the left.
$x + 2 = \pm\sqrt{8}$ $x = -2 \pm \sqrt{8} = -2 \pm 2\sqrt{2}$	Take the square root of both sides. Be sure to allow for both plus and minus.
$x = -2 + 2\sqrt{2}; \quad x = -2 - 2\sqrt{2}$	Solve for x.

Solve by completing the square:

1. $a^2 + 8a + 11 = 0$

2. $k^2 + 14k - 19 = 0$

3. $x^2 - 20x + 64 = 0$

Solve any method-

Example: $5 - 2x^2 = 13$, subtract '5' to both sides and divide by '-2': $x^2 = -4$, square root (with $\pm$) and you get $x = \pm 2i$

1) $-9 - 8x^2 = -209$

2) $7r^2 + 8 = 281$

3) $25m^2 - 8 = -7$

4) $2n^2 - 5 = -35$

Rationalize each denominator. When possible, simplify by reducing the resulting fraction.

Ex.. $\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{4}} = \frac{\sqrt{2}}{2}$

2. $\frac{2}{\sqrt{3}}$

3. $\frac{1}{\sqrt{7}}$